# Functions Of North America

*A field guide*

**Society for the Prevention of Cruelty to Functions**

David Sims, Publications Editor

This publication has been prepared by the Society for the Prevention of Cruelty to Functions (SPCF) in response to a recent outbreak of heartless cruelty to mathematical functions. During a recent National Functions Summit, it was decided that perhaps a booklet disseminated among the population would help raise awareness of the species of functions living on our continent and the types of horrors they experience. We will examine many different types of functions, and include information regarding their habitat (domain and range), forms and components, behavior, and natural enemies and limitations (asymptotes, holes.) The editor would like to extend thanks to the Graphics Staff at SPCF for not drawing a darn thing, and making him do it all by hand.

--David Sims
May, 2000



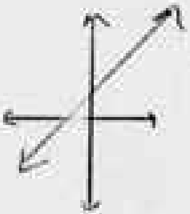
Excellent! A+

**Linear Functions**

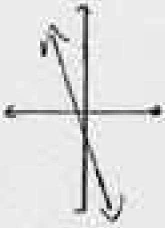
Basic form: $f(x)=mx+b$

m= slope
b= y-intercept

Linear functions can be found almost anywhere in North America. Most have a domain of all real numbers and a range of all real numbers. Lines are often mistaken for sticks or snakes and violently shaken, causing slope disorders. There are 4 various species:



$m>0$
D: $\mathcal{R}$
R: $\mathcal{R}$



$m<0$
D: $\mathcal{R}$
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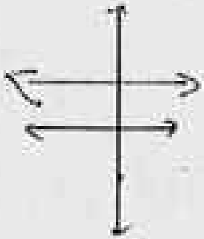
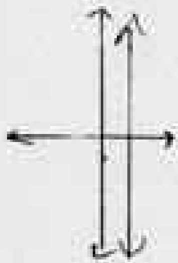


$m=0$
D: $\mathcal{R}$
R: b



m undefined
D: x
R: $\mathcal{R}$

Linear functions can also be written as

$$(y-b)=m(x-a)$$

where m is the slope and (a, b) is a point on the line.

Typical environment might include:

A BMW accelerates at 5 m/s. How fast is it going after 6 seconds? [This function produces a line]

Anything that appears as a straight line is probably a linear function. Linear Droopage, not to be confused with parabolas, is a sign of neglect and abuse; immediate hydration is required.

# Polynomial Functions

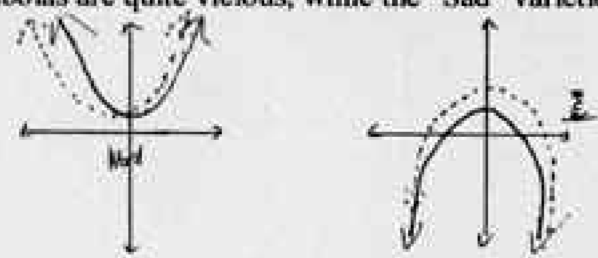
### *Quadratic*

Typical quadratic form: $f(x)=a(x-b)^2+c$

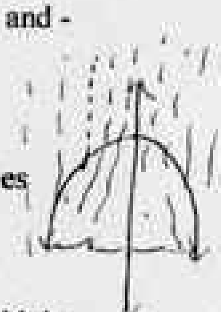
a= the "skinniness" or "fatness" of the parabola, an indication of the ideal weight of domestic functions

(b, c)= the vertex of the parabola (b affects the horizontal shift, and c the vertical shift.)

Species of parabolas are shown below. Contrary to popular math legend, "Smiling" parabolas are quite vicious, while the "Sad" varieties can be quite jovial.



Parabolas have a Domain=$\mathcal{R}$. If $a>0$, the Range $\geq$ c. If $a<0$, Range $\leq$ c. As $x \to \infty$ and $-\infty$, the parabola ends become more linear.



The roots (x-intercepts) of a quadratic formula can be found by using the quadratic formula. Often, the nature of parabolas suggests misuses such as games of horseshoes and impaling the vertex with a linear function to create an umbrella.

Typical environment:

Each year police increase the number of function abusers they catch (mostly high school students.) If, in the last 5 years, they've caught 1, 4, 9, 16, and 25 offenders, how many do you expect them to catch next year?

### *Cubic*

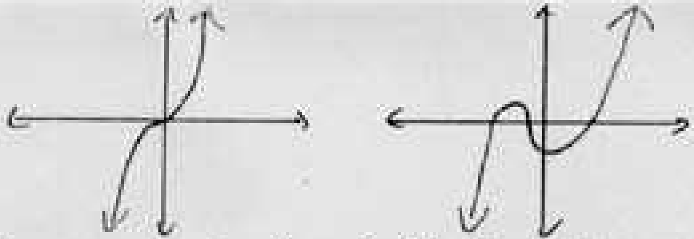
Typical form: $f(x)=(x-a)(x-b)(x-c)$

Where a, b, and c are roots of the function.

Good choice of what form to use here and above for the quadratic.

Cubes can be found in the whole Domain of $\mathcal{R}$ and the Range of $\mathcal{R}$. Depending on ancestry, they can cross the x-axis at most 3 times, as shown below.

Good!

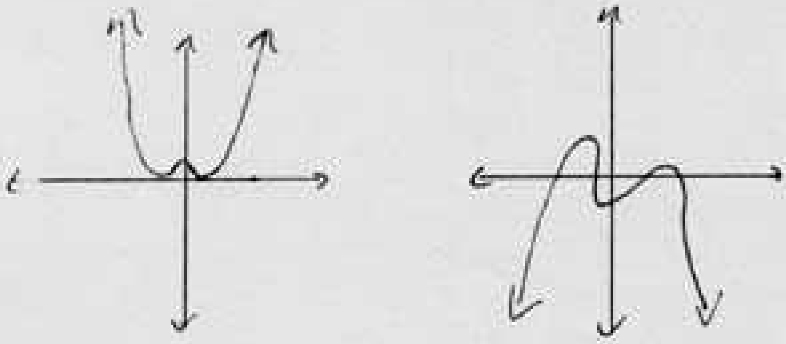


As $x \to -\infty$ or $x \to +\infty$, the ends of the function become more linear.

***Quartic***

$f(x)=(x-a)(x-b)(x-c)(x-d)$

Quartic functions are very similar to quadratic functions, except that they can have 4 zeros instead of 2. Their domains are $\mathcal{R}$, and their ranges are everything to the top or bottom of their most extreme vertex (see below.)



If all the powers in the function are even, the function is symmetric with respect to the y-axis. If some of the powers are mixed, they are not symmetric.

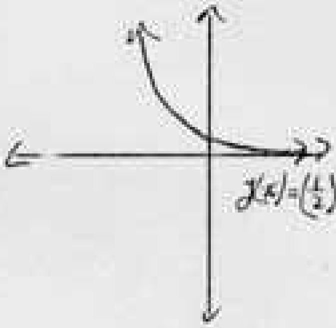
Quartic functions are thought to be games of cat's cradle, and are subjected to endless torture by small children. Parents **must** store functions of higher degrees in locked cabinets, in accordance with the Safe Math Act of 1997. This ensures that neither the user nor the delicate higher degree function is injured by carelessness.

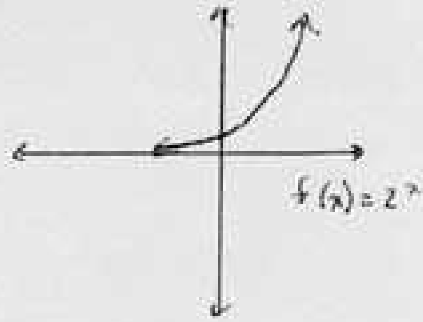
# Exponential Functions

Typical Form: $f(x)=a(b)^x$

a= the steepness of the upward slope; vertical stretch

b= base, a constant. As $b \to \infty$, the function appears more vertical.




Exponential functions are easy to spot in the wild, because they always have a y-intercept of 1. ($z^0=1$) Also, due to the nature of exponents, they can be found in the $\mathcal{R}$ domain, while the range depends on a. If a>0, R>0 and vice versa. Nearly all species commonly found have a>0, so y=0 is their asymptote. As shown above, $f(x)=2^x$ and $g(x)=(½)^x$ are reflections over the y-axis.

*Excellent!*

As $x \to {}^{+}\infty$ or ${}^{-}\infty$, the graphs become more linear and parallel to the axis.

Exponential functions are not to be used as shoehorns. Although they have the correct shape, ramming these functions into small places can cause damage to the central asymptotic system. Please encourage our efforts to boycott shoe stores that abuse them for this purpose.

Money Example:

The SPCF Trust Fund compounds interest according to the following formula:

$\$ = Pe^{rt}$. Where P is our capital investment and r is our 8% interest rate.

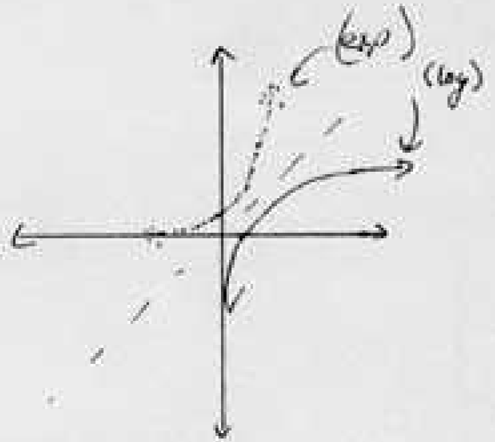
## Logarithmic Functions

Typical Form: $f(x)=a \log_b x$

Logarithms are the opposites of exponents; thus, the functions look very similar. Often, amateur function collectors will hold logarithmic functions at unnatural angles and mistake them for exponential functions. Not only is this asinine on the part of the collector, but the log may have an identity crisis and attempt suicidal linearization.

Because the exponent and the log functions are inverses of each other (reflected over $y=x$), one can find the logs in the exact opposite conditions as one would find an exponent. The Domain of logarithms is $x>0$, and the Range is $\mathcal{R}$. Logarithms have an x-intercept of 1, and an asymptote of $x=0$. It is therefore more understandable that reclining or sleeping logs appear to be exponents.

Very nice!




One of the most important reasons for supporting the prevention of logarithmic cruelty is the fact that we humans react to sound logarithmically. If we didn't, we could either hear very soft levels and be blown away by the vacuum cleaner's noise, or hear train whistles and not telephones.

Typical auditory environment:

Equations with sound levels have the form

$$\beta = 10 \log_{10} (I/I_0)$$

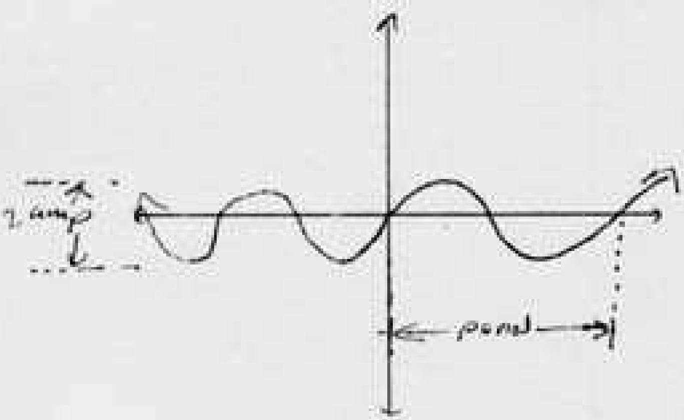
## Sinusoidal

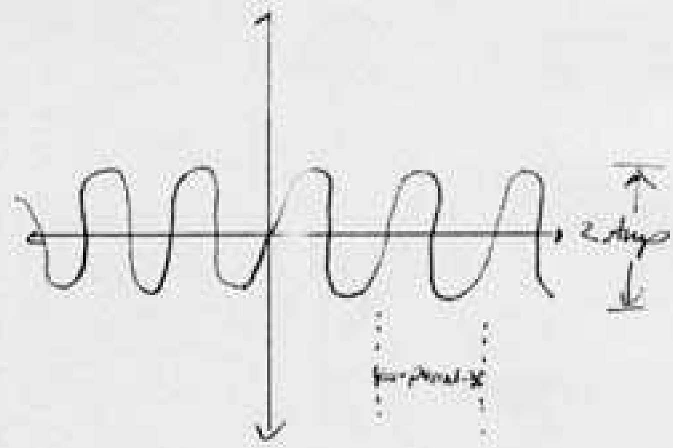
Typical form: $f(x)=a \sin(bx-c) + d$

> $|a|$ =amplitude of the function
> period= $2\pi/b$
> c= horizontal shift
> d= vertical shift

Sinusoidal functions are one of the most common trigonometric functions, and are periodic. They have a Domain of $\mathcal{R}$; However, the Range is restricted to the values between the amplitude. For $f(x)=\sin x$ , for example, the range is $-1 \leq y \leq 1$.

One of the most important characteristics of sinusoidal functions is the fact that they are periodic. Data with periods are usually fit to sine or cosine function models. (On the function black market, periodic precision is very valuable.) Here are some sine functions with their periods marked:





Do not use sines as rollercoasters or slides without ensuring they have the correct support. Unsupported sine functions forced to carry loads can lose their periodic pureness.

Typical environment:

True fact: For a person at rest, the velocity v of air flow during a respiratory cycle is

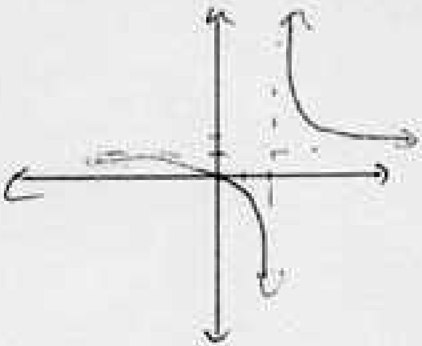
$$v= 0.85 \sin (\pi t/3)$$

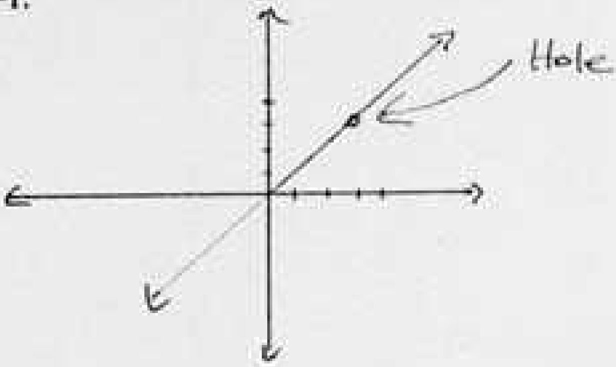
## Rational Functions

Rational functions are functions that can be written as the ratio of rational expressions, such as

$$f(x)=\frac{(x-a)}{(x-b)}$$

They are such advanced functions for amateurs because of their prevalent holes and asymptotes. In the above example $x \neq b$ to prevent the denominator from being 0. The function $f(x)= x/(x-2)$ has a vertical asymptote of $x=2$, and when we graph it, we see there is also a horizontal asymptote at $y=1$.






The Domain and Range of a rational function are entirely dependent on the specific function. They will include everything except the asymptotes and holes. (Holes occur in situations such as $f(x)= x(x-3)/(x-3)$. The (x-3)s will normally cancel, but there will be a hole at $x=3$.) This makes them harder to find in the wild, but also more vulnerable. The asymptotes of a rational function must be preserved at all costs. In addition, the holes of a rational function must be kept free of obstructions. One of the saddest sights in America today is the underground fetish of filling holes and asymptotes and watching the function wither and die, turning into a point. This type of behavior is illegal in all 50 states. We urge you to notify local authorities if misuse is observed.

Very nice!

### Typical environment:

Morris and Albert wash cars to raise money for function sanctuaries. It takes Albert 2 minutes longer to wash a car than Morris. Together they wash 9 cars per hour. How long does it take Morris to wash a car?

$60/M + 60/(M+3) = 9$

## Power Functions

Typical form: $f(x) = ax^b$

a= skinniness or fatness of the graph

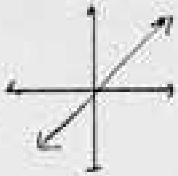
b= many interesting things, to be discussed below.

First off, it should be said that power functions are very dangerous by nature. These functions are cute near the origin, but as one gets further along the axis they become too big to handle. Often times they are enclosed in cages way to small for their size. Function preserves exist to provide a sanctuary for mistreated functions, where they can have the room they need to grow and expand.

If $b=1$, it looks like this:

D: $\mathcal{R}$

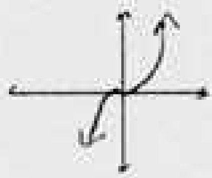
R: $\mathcal{R}$



If $b>1$ and odd, it has this shape:

D: $\mathcal{R}$

R: $\mathcal{R}$



If $b>1$ and even, it has this shape:

D: $\mathcal{R}$

R: $y \geq 0$

If $0<b<1$, it looks like this:

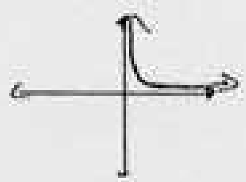
D: $x \geq 0$

R: $y \geq 0$

If $-1<b<0$, it looks like this:

D: $x>0$, asymptote at $x=0$

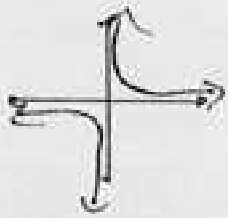
R: $y>0$, asymptote at $y=0$



If $b \leq -1$ and odd, it looks like this:

D: asymptote at $x=0$

R: asymptote at $y=0$

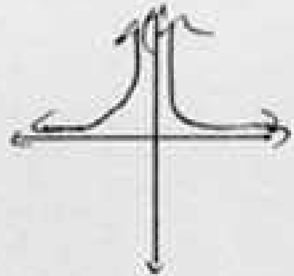


wow!

If b≤-1 and even, it looks like this:

D: asymptote at x=0

R: asymptote at y=0



As you can see, power functions are probably the most varied of all of the functions we've encountered.

Typical environment:

A generous donor will square the amount of any donation made to the Society for the Prevention of Cruelty to Functions. If a contribution is made for $100, how much will the SPCF receive?

$100^2$

# Function of the Year

*For this year, we decided to choose the* ***absolute value function*** *as our SPCF Function of the Year. One afternoon a ferocious teenager was using an absolute value function for a boomerang outside of SPCF HQ. This really hit home with our staff, and we are devoting this page to absolute values.*

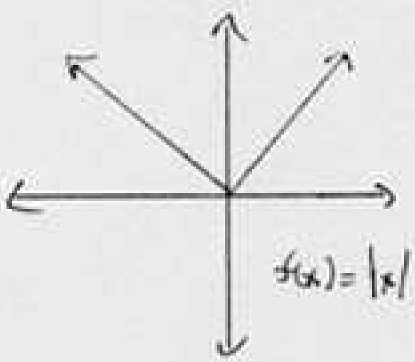
Typical form: $f(x) = a|x-b|+c$

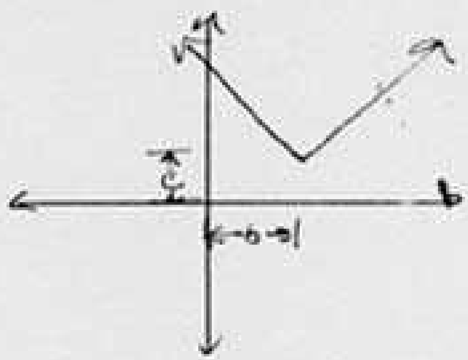
a=vertical stretch

b=horizontal shift of b

c=vertical shift of c

By the nature of the meaning of absolute value, the Domain=$\mathcal{R}$ and the Range ≥0, or whatever c happens to be. The graph is symmetric over the y-axis. Here are some examples of absolute value graphs:



Absolute value functions have many practical applications when talking about distances, because, like distances, they are always positive. And please, don't use them as boomerangs.

If you would like more information regarding the Society for the Prevention of Cruelty to Functions, including membership information and donation information, please send inquires to:

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1210 W. Springfield
Urbana, IL 61801

and send money directly to:

David Sims, editor
University High School
1212 W. Springfield
Urbana, IL 61801

Thank you for your support of the SPCF!

✂ ..........................................................................................

NOMINATION FOR NEXT *FUNCTION OF THE YEAR* (VOTING MEMBERS ONLY)

I nominate ______________________________ for the SPCF *Function of the Year* award. It deserves this award because, in addition to being a beautiful mathematical model, it ______________________________

Additional Comments:

Signed: Date: